

Recall to Reasoning: Mathematics Foundations for Fluency

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Catholic
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Beyond numeracy

Why Effective Mathematics Teaching Matters

Mathematics is a structured discipline that fosters problem-solving and logical reasoning.

Our end goal create confident, capable students who have a strong mathematical sense and disposition through success in the subject.

We need to do it through research-based instructional methods.



Effective Mathematics Teaching

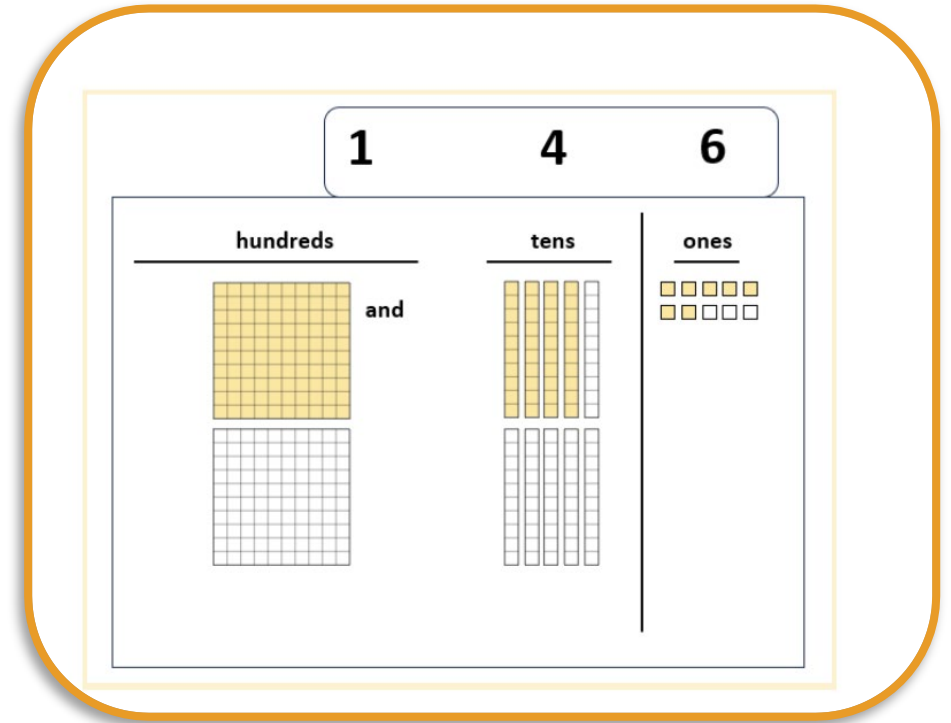
Is Explicitly Taught

Effective mathematics instruction should incorporate explicit teaching, formal academic language, and multiple representations and models.

Laying a strong foundation for number sense.

Mathematics strengthens logical reasoning and problem-solving.

Effective instruction builds neural pathways that support conceptual understanding.



Acquisition → Accuracy → Fluency → Generalise

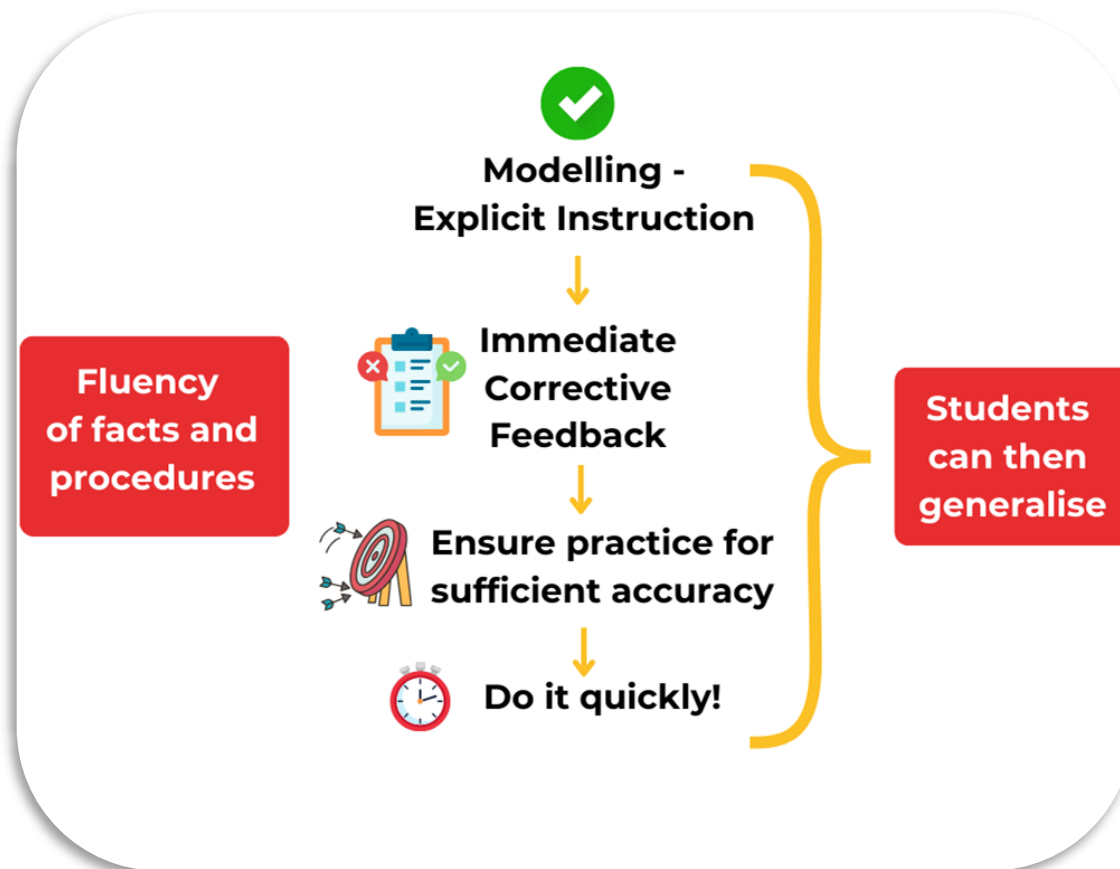
The Model for Learning

Practice to problem solving!

Retrieval is the super-power of learning.

Success and achievement in mathematics is no different then learning any other skill.

Needs to be spaced, purposeful and with increasing effort to strengthen memory and learning.



Power is in the practice

The how and when of sufficient practice

It is only with sufficient practice that students have the knowledge links that can then be used to work not just in problem solving

But the open-ended unfamiliar task

- 10 A jar is half full of smarties.
When 50 smarties are added the jar is $\frac{3}{4}$ full.
How many smarties will there be in a full jar?

50 150 100 200

☐ ☐ ☐ ☐

Ainslie ran in a race. After she had run 3.2 km she still had three quarters of the race to go. What was the total length of the race?

1.6 km 6.4 km 6 km 12.8 km

☐ ☐ ☐ ☐

Shade one bubble.



Systematically Sequenced

Mathematics is cumulative

- 'Ladder effect' due to its hierarchical cumulative nature
- Any concept we teach depends on a whole bunch of other concepts that they should have learned beforehand.
- Imperative that students have a great foundation



← *algebraic symbolism*

← *equality*

← *order of operations*

← *proportion*

← *ratios*

← *understanding number*
(times tables)

Intervention at all levels

Across the grades

Early success is predictive

Intervene at the point at error

Can't do addition and subtraction without counting

Opening the door for mathematics – points of intervention

Embedding a 'Maths Block'.

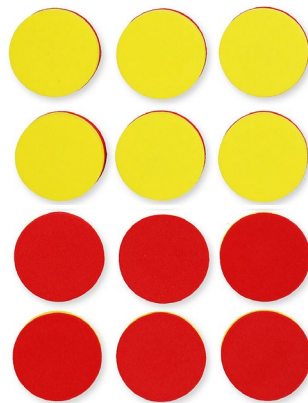
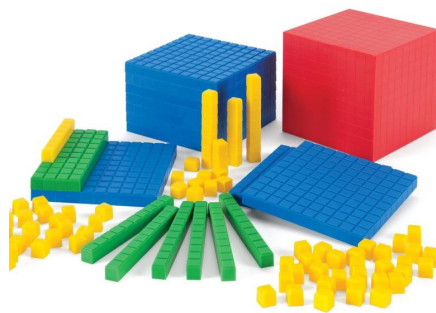
Does maths intervention have the same resources as reading?



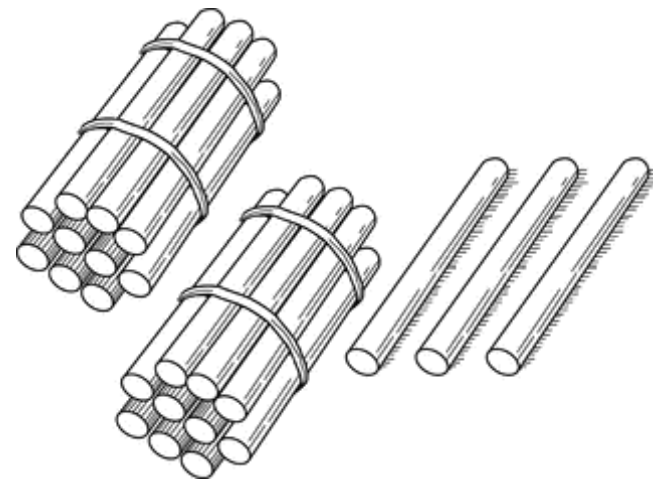
Make links between concrete, representations and abstract to develop the concepts

Multiple representations increase conceptual understanding

23



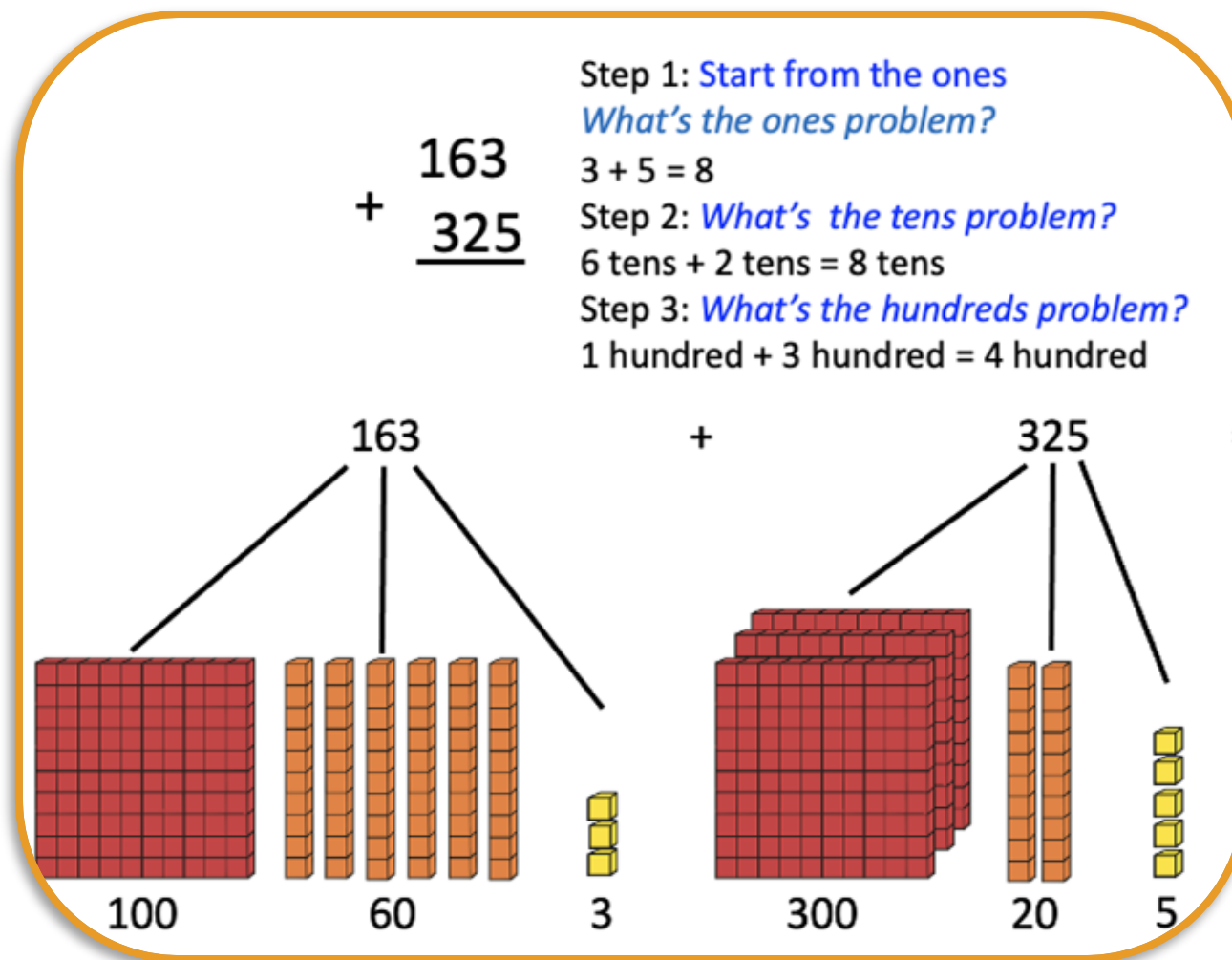
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Concepts are connected to Procedures

Two-way relationship

Teaching conceptual and procedural knowledge together helps strengthen each other over time.



Adaptive Reasoning

Using a variety of Models

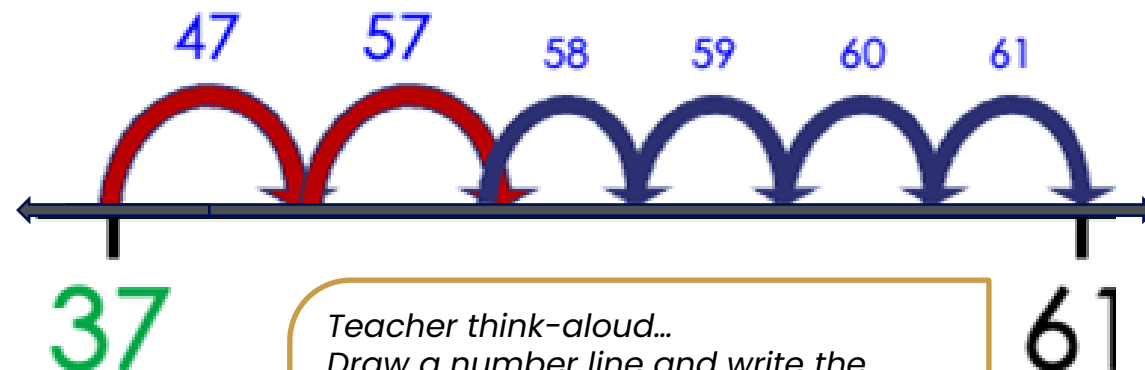
The high yield effect of number lines

Increase understanding for magnitude

Extremely important when learning fractions

Can be used as a variety model to support mental maths and computation..

$$37 + 24 = 61$$



Teacher think-aloud...
Draw a number line and write the larger number on the left end of the line.
How many am I adding on?
Jump the tens
Jump the ones

Adaptive Reasoning

Models for mental computation

Add numbers using an empty line:

3 286

Fit for purpose!

2027 =

Be wary...

Addition using place value



$$234 + 567$$

Step 1: expand each number

Step 2: add the hundreds

Steps 3: add the tens

Step 4: add the ones

Step 5: combine all the totals

Fit for purpose!

Step 1: $234 = 200 + 30 + 4$

$$567 = 500 + 60 + 7$$

$$789 = 700 + 80 + 9$$

$$500 + 700 = 1\ 400$$

$$60 + 80 = 170$$

$$4 + 7 + 9 = 20$$

$$1\ 400 + 170 + 20 = 1\ 590$$

Mork travels the same distance to and from the games shop every day. His house is 16km from the games shop. How far would he travel in 6 days?

The length of the Bruno's electric bike is three times less than the length of Brendan's car. If the car is 18 meters long, how long is Bruno's bike?

Understand
Plan
Solve



Systematically Sequenced

Mathematics is cumulative

Begin with number stories (pictorial representations)

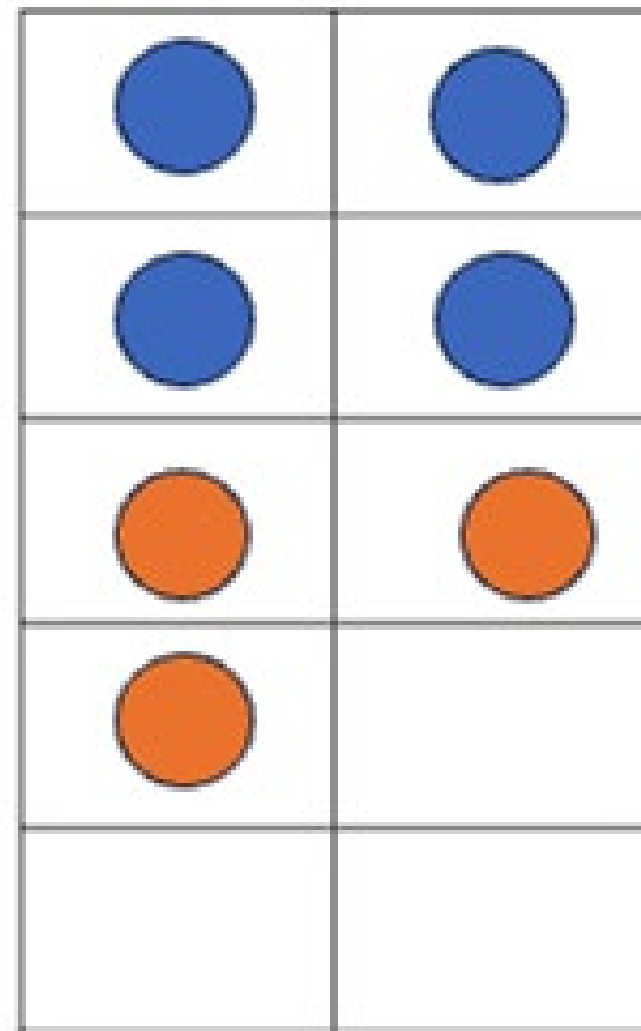
There were 3 sheep in the field. 3 more sheep joined in the field. How many sheep altogether.

Unifix blocks can be used to model

But quickly move to tens frame representations with coloured counters

With clear introduction of symbols and algorithm shown early as the symbolic representation of the picture.

$$+ \begin{array}{r} 4 \\ 3 \\ 7 \end{array}$$



Teaching Algorithms

Teacher micro scripts keep consistent language

A high level of variance between classes in language and methods has a knock-on effect of students moving up the grades without solid prior knowledge to build new knowledge on.

$$\begin{array}{r} 43 \\ - 15 \\ \hline \end{array}$$

$$\begin{array}{r} 313 \\ \cancel{4} \cancel{3} \\ - 15 \\ \hline \end{array}$$

$$\begin{array}{r} 313 \\ \cancel{4} \cancel{3} \\ - 15 \\ \hline 8 \end{array}$$

$$\begin{array}{r} 313 \\ \cancel{4} \cancel{3} \\ - 15 \\ \hline 28 \end{array}$$

*I have 3 ones, can I take away 5 ones?
(signal)*

No, regroup

*I have 13 ones, can I take away 5 ones?
(signal)*

Yes, do it!

13 ones take away 5 ones equal 8 ones

I have 3 tens. Can I take away 1 ten? (signal)

Yes, do it!

3 tens take away 1 ten equals 2 tens

Retrieval

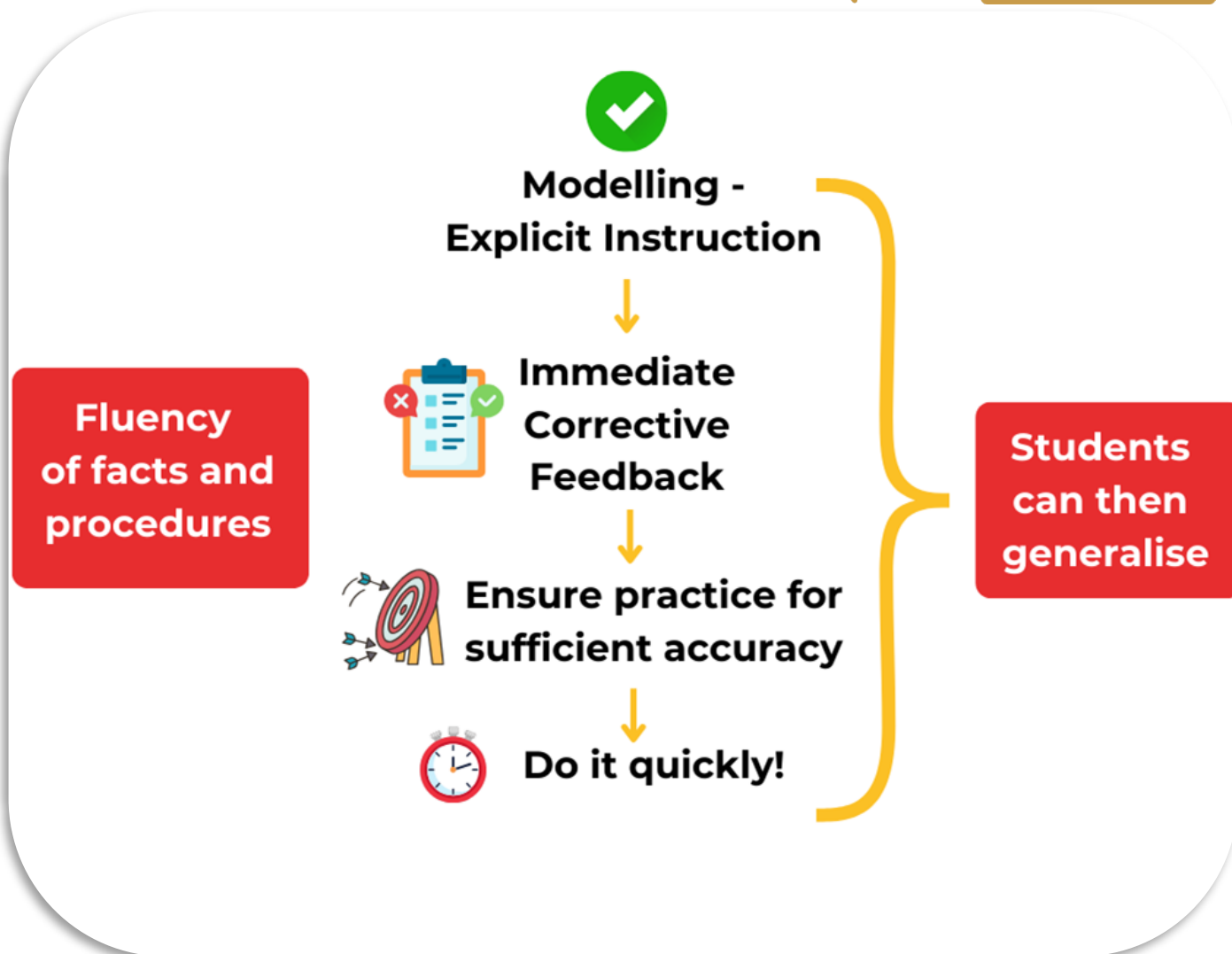
The Model for Learning – How

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Practice to problem solving!



Focus on the Language of Mathematics

If you are teacher of Maths you are a teacher of language

Significant correlation to vocabulary and performance

Unique predictor of word problems

Hundreds of terms students need to know by Year 3 more than 300, across K-8 – 1200 unique vocabulary terms.

Numerator, polygon, factor, symbol, equal, plus, numerator, denominator...

Difference (*difference* in number *difference* between seasons)



Beyond Word Walls

How useful?

Word walls are only useful if referred to often and used as anchor charts...

Vocabulary word / student definition / visual representation

A tell B how to solve a problem, using 3 terms from the vocabulary word wall

Matching games – visual to word to definition

English a 2nd or 3rd language then teach one explicitly

Addition is the same as:



- | | |
|------------|--------------|
| - add | - plus |
| - total | - altogether |
| - sum | - combine |
| - join | - how many |
| - increase | - in all |

Language for Purpose

Maths vocabulary in use

What's the product of 6 and 9?

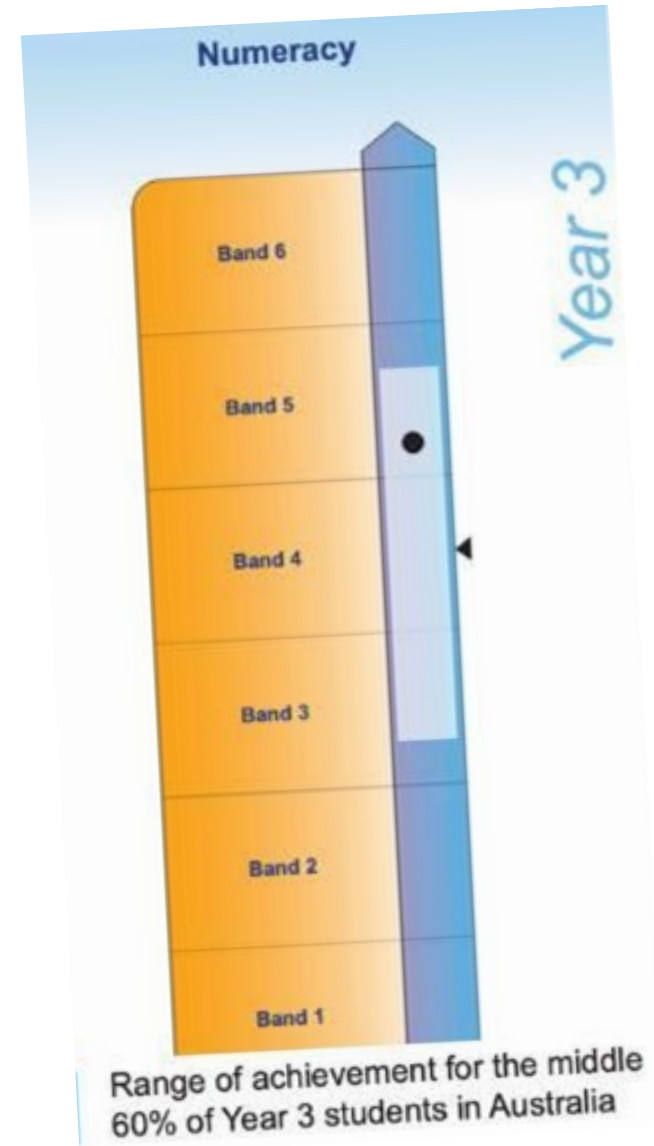
What is 54 shared between 6?

Find the sum on 9 and 8?

How many times can 6 go into 48?

What's the difference between 12 and 1?

Find the quotient of 40 and 5?



Dynamic instructional routines

Highly interactive routines

- Management of student responding – high frequency, using signals
- Keep the pace 'perky' and students actively responding (verbal, written, actions)
- Whole class responses ensure all students do the thinking and allows the teacher to monitor student knowledge and give responsive feedback
- Management of attentional control – routines for focus





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